Analysis and Computation

Charles R. Kiss

December 10, 2009

Abstract

The metric of a time-dependent inverse power function with a positive constant is examined within a closed unit interval.

$$F(r,t) = -\frac{k \cdot t}{(r)^m} + \frac{k}{(b)^m \cdot t} \qquad a \leq r \leq a+1, \quad 0 < t \leq 1, \quad k = constant. \tag{1}$$

As the derivatives and function values change inversely with time -at the upper and lower limits of the domain, the unit interval remains unchanged. The upper and lower limits of the domain are determined simultaneously by the positive term, dependent on t, and the value of the total integral which must always be zero. As the positive integral must always equal and oppose the negative integral, each is set to unity and negative unity repectively.

Because solutions involve equations of the form m^m, an algorithm is used to determine the values of a, b, and m for values of t.

Part I

Definitions

1 Function Values

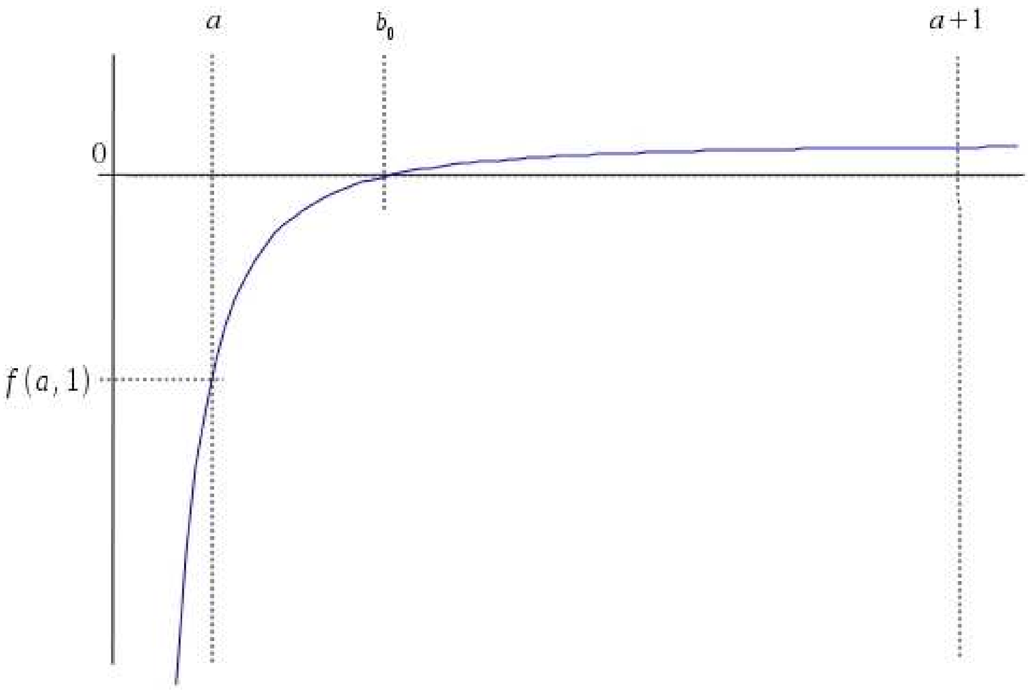


Fig.1

Given the function[Fig.1]:

$$f(r,t) = -\frac{k \cdot t}{(r)^m} + \frac{k}{(b)^m \cdot t} \qquad a < r < a+1, \qquad 0 < t \leq 1 \tag{2}$$

Define the negative maximum at the lower limit of the function $r = a$, $t = 1$

$$f(a,1) = -A \tag{3}$$

Solve for a,

$$a = \left(\frac{kt^2(b)^m}{At(b)^m + k} \right)^{\frac{1}{m}} \tag{4}$$

If $A = 0$, solve for r

$$r = b \cdot t^{\frac{2}{m}} \tag{5}$$

If$A = 0$ and $t = 1$, for all m:

$$r = b \tag{6}$$

If $k \ll A \cdot (b)^m$ [see Section II.3]:

$$a = \left(\frac{kt}{A} \right)^{\frac{1}{m}} \tag{7}$$

and

$$A = \frac{kt}{a^m} \tag{8}$$

In the special case that $t = 1$, $m = 2$, and $k \ll A \cdot (b)^m$, equation (7) takes the form of the negative inverse square function for $r = a$.

$$f(r, 1) = -\frac{k}{r^2}$$

2 Derivative Values

Define maximum value of the derivative at $r = a$, $t = 1$

$$f'(a, 1) = B \tag{9}$$

Solve for r,

$$r = \left(\frac{mkt}{B} \right)^{\frac{1}{m+1}} \tag{10}$$

If $r = a$,

$$a = \left(\frac{mkt}{B}\right)^{\frac{1}{m+1}} \tag{11}$$

2.1 Approximation Excercises

If both the function value and the derivative changed linearly in some time interval, equation (7) instead would become:

$$a = \left(\frac{k \cdot t}{A}\right)^{\frac{1}{m}} \to \left(\frac{k \cdot t}{A \cdot t}\right)^{\frac{1}{m}} = \left(\frac{k}{A}\right)^{\frac{1}{m}} \tag{12}$$

as equation (11) would become:

$$a = \left(\frac{mkt}{B}\right)^{\frac{1}{m+1}} \to \left(\frac{mk \cdot t}{B \cdot t}\right)^{\frac{1}{m+1}} = \left(\frac{mk}{B}\right)^{\frac{1}{m+1}} \tag{13}$$

so that:

$$\left(\frac{k}{A}\right)^{\frac{1}{m}} = \left(\frac{mk}{B}\right)^{\frac{1}{m+1}} \tag{14}$$

or:

$$k = m^m \left(\frac{A^{m+1}}{B^m}\right) \tag{15}$$

As a result equation (15) is independent of time.

Otherwise using the approximation, equation (7), as discussed in Section II, is

$$k \cdot t = m^m \left(\frac{A^{m+1}}{B^m}\right) \tag{16}$$

Again a more special case is where, $t = 1$, and so $m = 2$, in which case:

$$k = \frac{4A^3}{B^2} \tag{17}$$

3 Integral Values

3.1 Integral from $r = a$ to $r = b_0$ is negative unity.

The b_0 is defined as the point where the the function $F(r, t) = 0$, therefore according to (5),

$$b_0 = b \cdot t^{\frac{2}{m}} \tag{18}$$

$$\int_{a}^{b_0} -\frac{k \cdot t}{(r)^m} + \frac{k}{(b)^m \cdot t} dr = \int_{a}^{b \cdot t^{\frac{2}{m}}} -\frac{k \cdot t}{(r)^m} + \frac{k}{(b)^m \cdot t} dr = -1 \tag{19}$$

Rendering the integration:

$$\left[\frac{k \cdot t}{(m-1)(r)^{m-1}} + \frac{k \cdot r}{(b)^m \cdot t}\right]_a^{b_0} = -1$$

$$\frac{(m-1)(b_0)^{m-1}(a)^{m-1}(b_0 - a) + t^2 (b)^m [(a)^{m-1} - (b_0)^{m-1}]}{t \cdot (m-1)(b_0)^{m-1}(b)^m (a)^{m-1}} = -\frac{1}{k} \tag{20}$$

Simplifying, by equation (18), that $(b)^m \cdot t^2 = b_0$:

$$\frac{(m-1)(a)^m (b_0)^{m-1}[(b_0)^{-1} - (a)^{-1}] + [(b_0)^{m-1} - (a)^{m-1}]}{(m-1)(a)^{m-1}} = \frac{1}{kt} \tag{21}$$

3.2 Integral from $r = b_0$ to $r = a + 1$ is positive unity.

$$\int_{b_0}^{a+1} -\frac{k \cdot t}{(r)^m} + \frac{k}{(b)^m \cdot t} dr = 1 \tag{22}$$

3.3 Integral from $r = a$ to $r = a + 1$ is zero.

$$\int_{a}^{a+1} -\frac{k \cdot t}{(r)^m} + \frac{k}{(b)^m \cdot t} dr = 0 \tag{23}$$

Rendering the intergration:

$$\left[\frac{k \cdot t}{(m-1)(r)^{m-1}} + \frac{k \cdot r}{(b)^m \cdot t}\right]_a^{a+1} = 0$$

$$\frac{(m-1)(a+1)^{m-1}}{\left[\left(\frac{a+1}{a}\right)^{m-1} - 1\right]} = (b)^m \cdot t^2 \tag{24}$$

or, by equation (18), that $\left[(b)^m \cdot t^2\right]^{\frac{1}{m}} = b \cdot t^{\frac{2}{m}}$:

$$\left[\frac{(m-1)(a+1)^{m-1}}{\left(\frac{a+1}{a}\right)^{m-1} - 1}\right]^{\frac{1}{m}} = b_0 \tag{25}$$

A quick check for this equation is the special case where $t = 1$, and $m = 2$; because it is known:

$$(a)(a+1) = (b_0)^2 \tag{26}$$

4 Changes in the function $F(r,t)$ with time

The derivatives and function values of adjacent metric spaces at congruent times are equal. The integral between the upper and lower limits of the domain in congruent metric spaces and congruent times are equal.

$$F(r,t) = -\frac{k \cdot t}{(r)^m} + \frac{k}{(b)^m \cdot t}$$

The subscript of r denotes the metric scale of the r-space. The subscript of t denotes the metric of the t-space. So that the metric,

$$r_n \neq r_{n+1}$$

but,

$$r_n \propto r_{n+1}$$

and,

$$t_n \neq t_{n+1}$$

but,

$$t_n = t_{n+1} - 1$$

4.1 Changes looking forward in time: Adjacent metric space, Congruent time:

$$f(r_n, t_n) \rightarrow g(r_{n+1}, t_{n+1}) \rightarrow h(r_{n+2}, t_{n+2})$$

$$f(r_n, t_n) \rightarrow f(r_n, t_{n+1}) \Leftrightarrow g(r_{n+1}, t_{n+1}) \tag{27}$$

$$g(r_{n+1}, t_{n+1}) \rightarrow g(r_{n+1}, t_{n+2}) \Leftrightarrow h(r_{n+2}, t_{n+2}) \tag{28}$$

4.2 Changes looking backward in time: the same but reversed:

$$h(r, t_n) \to g(r, t_{n-1}) \to f(r, t_{n-2})$$

$$h(r_n, t_n) \to h(r_n, t_{n-1}) \Leftrightarrow g(r_{n-1}, t_{n-1}) \tag{29}$$

$$g(r_{n-1}, t_{n-1}) \to g(r_{n-1}, t_{n-2}) \Leftrightarrow f(r_{n-2}, t_{n-2}) \tag{30}$$

5 Rules

5.1 Equalities forward in time

5.1.1 Changes in Function Values

$$F(a_M, t_{M+1}) = F(a_{M+1}, t_{M+1}) \tag{31}$$

5.1.2 Changes in Derivative Values

$$F'(a_M, t_{M+1}) = F'(a_{M+1}, t_{M+1}) \tag{32}$$

5.1.3 No Changes in the Integral: Congruent metric space, congruent time:

$$\int_{a_M}^{a_M+1} F(r_M, t_M) = f(r_n, t_n) = g(r_{n+1}, t_{n+1}) = ... = 0 \tag{33}$$

The symbol M defines the metric on the r-space, whether it is the n-metric, the $n+1$-metric, etc. For clarity, the upper limit of the integral, as defined in equation (22), is written $a_M + 1$, which is a numeral, and not to be confused with the writing a_{M+1} which is used to define a metric adjacent in time to a metric a_M.

5.2 Equalities backward in time: quick look.

5.2.1 Changes in Function Values

$$F(a_M, t_{M-1}) = F(a_{M-1}, t_{M-1}) \tag{34}$$

5.2.2 Changes in Derivative Values

$$F'(a_M, t_{M-1}) = F'(a_{M-1}, t_{M-1}) \tag{35}$$

5.2.3 No Changes in the Integral

$$\int_{a_M}^{a_M+1} F(r_M, t_M) = f(r_n, t_n) = g(r_{n+1}, t_{n+1}) = ... = 0 \tag{36}$$

Part II

Quantification

$$G = 6.67 \times 10^{-11} \frac{m^3}{kg \cdot sec^2}$$

$$M_\ominus = 4 \times 10^{40} kg$$

$$M_\odot = 2 \times 10^{30} kg$$

$$1 lightyear = 9.4605284 \times 10^{15} meters$$

$$1 kiloParsec = 3.08568025 \times 10^{19} meters$$

$$a = \frac{2GM_\ominus}{c^2}$$

$$b_0 = \frac{GM_\ominus}{v^2}$$

1. Calculating the average distance between galaxies

A simple case when $t = 1$, and so $m = 2$, using equation (26), the average distance between galaxies, defined as unity, can be determined.

$$1 = \frac{1}{2}\frac{GM_{\ominus}}{c^2v^4}\left(c^4 - 4v^4\right)$$

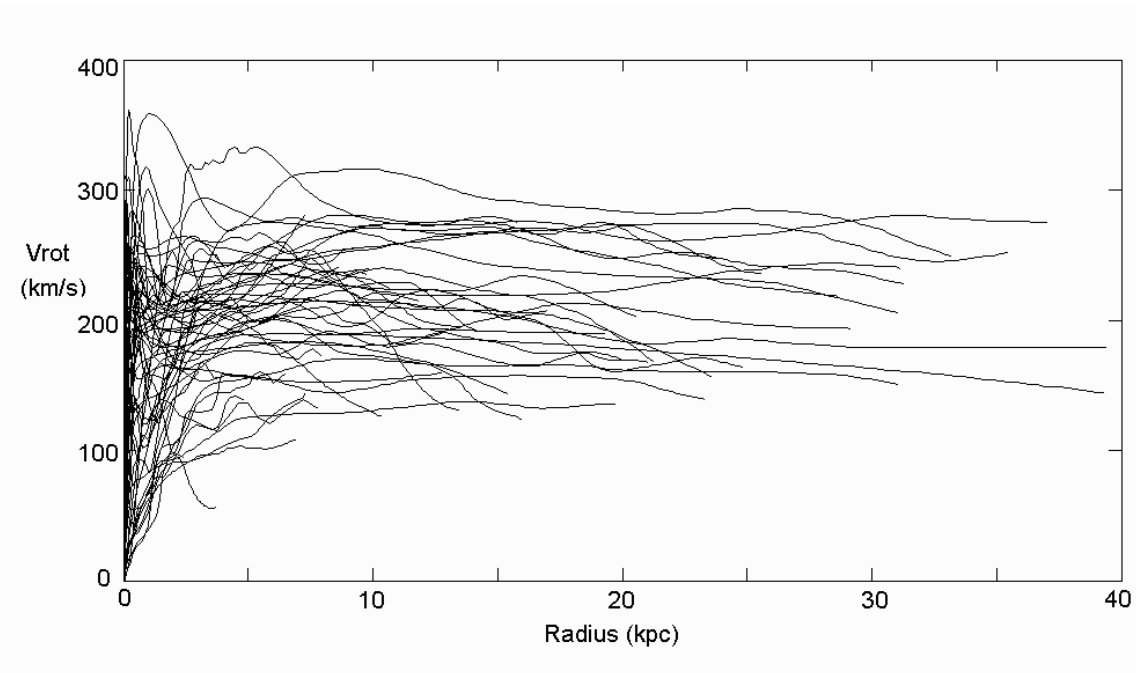
or

$$1 = \frac{1}{2}\frac{b_0}{c^2v^2}(c^4 - 4v^4) \quad (37)$$

Approximating by removing the term$-4v^4$, which is over eleven orders of magnitude lower than c^4.

$$1 = \frac{1}{2}b_0\left(\frac{c^2}{v^2}\right) \quad (38)$$

Where $M_{\ominus}$ is the mass of an average galaxy within the boundary b_0 where the rotation velocity is v^2. For an example, using the chart below: $v \approxeq 220\frac{km}{s}$ and $b_0 \approxeq 2kPc$.



The calculation yields.

$$1 = \frac{1}{2}\left(6 \times 10^{19}\right)\left(\frac{3 \times 10^8}{2.2 \times 10^5}\right)^2 meters$$

$$= 6 \times 10^{25} meters \approxeq 6 \times 10^{9} lightyears \approxeq 2 \times 10^{6} kPc$$

$$\approxeq 50,000\, galaxy\, diameters$$

These figure of an average of 50,000 galaxy diameters between average sized galaxies represents the density today, at $t = 1$, and not the density of what is visible where $t < 1$.

The density of space would therefore be:

$$\frac{2M_{\ominus}}{\frac{4}{3}\pi r^3} = \frac{2(4 \times 10^{40} kg)}{\frac{4}{3}\pi(6 \times 10^{25} meters)^3} = \frac{8 \times 10^{40}}{9 \times 10^{77}} \approxeq 1 \times 10^{-37} kg/m^3 = 1 \times 10^{-40} g/cm^3$$

This is equivalent to about one atom of hydrogen for every 2km volume.

2. Calculating the average distance between stars

Rather silly but fun, nonetheless, if the universe were composed of individual stars the size of the Sun, using the velocity and distance of a bound planet such as Mercury for v , and the semi-major axis b_0, ignoring relativistic effects, in equation (38):

$$1 = \frac{1}{2}(5.8 \times 10^{10} m)\left(\frac{3.0 \times 10^{8} m/sec}{4.8 \times 10^{4} m/sec}\right)^2 \approxeq 1 \times 10^{18} meters \approxeq 100 lightyears$$

Similarly, a pencil dot of diameter of 0.1mm every 5 meters.

3. The elimination of *k* in the denominator of equation (10) and the resulting contsraint on the usefulness of the approximation.

Given equation (10):

$$a = \left(\frac{kt^2(b)^m}{At(b)^m + k}\right)^{\frac{1}{m}}$$

Comparing the magnitude of k, if it is very small relative to $At(b)^m$:

$$At(b)^m >> k$$

Given,

$$b = \frac{GM_\ominus}{v^2}$$

and,

$$k = GM_\ominus$$

we know, as defined, that when $t = 1$, $m = 2$, therefore :

$$A \left(\frac{GM_\ominus}{v^2} \right)^2 >> GM_\ominus$$

–

or, since $A > 1$, simplify,

$$\frac{GM_\ominus}{v^4} >> 1$$

or,

$$\frac{b}{v^2} >> 1$$

From the chart above, using $v \approxeq 220km/sec$, and $b \approxeq 2kPc \approxeq 6 \times 10^{13}km$,

$$\frac{6 \times 10^{13}km}{48400km^2/sec^2} > 1 \times 10^9 >> 1$$

Therefore it can be expected that the approximation is a useful one provided that $t \approxeq 1$, and $m \approxeq 2$; how closely for the purposes of this examination can be determined by either t or m alone since each is dependent on the other. Focusing on t then, limiting the time interval to between that interval required to contain three or four galactic rotations, the sample of data of sufficient size, perhaps, to properly analyze the behavior of the orbits of these stars about the galactic center over the last one billion years, or less than 10% the accepted age of the Universe.

The closer the domain of the function is limited to a smaller interval near t, for example, $0.99 \leq t \leq 1$, the more accurate is the approximation, up to 9 orders of magnitude.